UNIT = 01

INTRODUCTION

LIMITATIONS OF ANALOG SIGNAL PROCESSING:-

1. Unwanted Noise is Recording
2. If we x'mit data at long distance then unwanted disturbance is there
3. Generation loss.
4. Less Immune to noise
5. More likely to get affected. Reducing accuracy.
6. Requires high Quality processing which in turn demands costly hardware.
7. High power Requirements
8. costly storage Requirements due to more data.
9. Limited processing speed (10) Lack of flexibility
11. High cost 12. Difficulty in implementing useful operations.

→ signal
→ system
→ analog / discrete / digital

DIGITAL SIGNAL PROCESSING:

DIGITAL:- electronic technology that generates, stores and processes data in terms of two states: +ve and non-positive. or in terms of 0 and 1.

SIGNAL:- is an electric current or electromagnetic field used to convey data from one place to another

simplest form of signal is DC that is switched on & off. this is the principle by which the early telegraph worked.

PROCESSING:- movement of data towards a known goal by passing it through series of action

NOTES

DSP - To process s/n digitally

DIGITAL SIGNAL PROCESSING

The process of converting analog x'ssion into digital signals

OR

It is the process of analysing and modifying a signal to optimise or improve its efficiency or performance. It involves applying various mathematical and computational algorithms to analog and digital signals to produce a signal that's of higher quality than the original signal.

ANALOG SIGNAL PROCESSING

Continuous time input signal $x(t)$ → [Analog system or Analog signal processor] → Continuous time output signal $y(t)$

DIGITAL SIGNAL PROCESSOR

Discrete time input signal $x(n)$ → [Digital s/n or Digital s/n processor] → Discrete-time output signal $y(n)$

BASIC ELEMENTS OF DSP

input analog signal → [ADC] → i/p digital s/n → [DSP] → o/p digital signal → [DAC] → output analog signal

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NOTES

ADVANTAGES OF DSP OVER ASP

① FLEXIBILITY:-

DSP systems are flexible.

→ Flexibility means system can be reconfigured for some other operation by simply changing the software program.

eg:- High pass digital filter can be changed to low pass digital filter by simply changing the s/w program.

In case of ASP it is not possible. In ASP you have to change whole hardware part which is complicated.

② ACCURACY:-

In ASP, accuracy is less. becoz analog system suffers from component tolerances, their breakdown etc. Hence difficult to attain accuracy.

In DSP the accuracy is decided by resolution of A/D convertor, no. of bits to represent digital data etc. these factors are possible to control DSP systems to get high accuracy.

③ EASY STORAGE:-

Digital signals easily stored on storage media such as magnetic tapes, disks etc.

→ Remote processing of Digital s/n is possible.

Analog s/ns suffer from the storage problems like noise, distortion etc.

④ MATHAMATICAL PROCESSING:-

NOTES

Mathamatical operations accurately performed on digital s/ns as compared to analog. s/ns.

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~~q being the amount of charge that can be stored in a capacitor of capacitance C against a potential difference of V volts. i.e~~

$$C = \frac{q}{V}$$

(5) COST:-

when there is large complexity in the application, then DSP systems are cheaper as comp to ASP systems.

- software control algorithms can be complex but it can be implemented accurately with less efforts.

(6) REPEATABILITY:-

processing of signals is completely digital in DSP systems. Hence performance is repeatable.

eg:- Low pass filtering operation performed by digital filters today will exactly same even after ten years.

But it may deteriorate in analog systems becoz of noise effect and life of components etc.

(7) ADAPTABILITY:-

DSP systems are easily upgradable since they are software controlled.

- easily upgradation not possible in analog systems.

(8) SIZE AND RELIABILITY:-

DSP systems are small in size, more reliable and less expensive as compared to analog systems.

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NOTES

DISADVANTAGES OF DSP SYSTEMS:-

① DSP systems are expensive for small applications. Hence the selection is done on the basis of cost complexity and performance.

②

SIGNAL

It may be a f=n of time, temperature, position, pressure, distance etc.

Daily life used signals — music, speech, picture and video signals.

Signal is a f=n of one or more independent variables which contains some Information.

In electric sence s/n can be voltage or current.

voltage or current is the f=n of time as an ind-ependent variable.

SUNDAY 6

In daily life we used several s/ns such as Radio s/ns, T.V s/n, Computer s/n etc.

Signal

one-dimentional ↓ when f=n depends on single variable. eg:- speech s/n where amp. varies with time.

multi-dimentional ↓ when f=n depends on 2 or more variables. eg:- image - signal with horizontal and vertical coordinates.

NOTES

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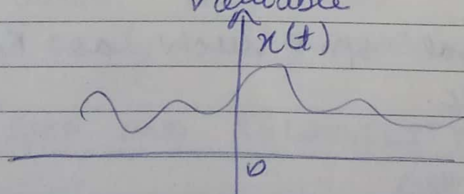
CLASSIFICATION OF SIGNALS:

Continuous - time signals

- defined continuously in the time domain
- for continuous time sgns independent variable is t.
- so sgn represented as $x(t)$
- sgn of continuous amp. and time is known for analog sgns.
- sgn will have value at every instant of time
- continuous time sgn is represented as $x(t)$ where
  x = represents shape
  t = " time which is variable



Discrete - time signals

- defined only at certain time - instants.
- independent variable is n
→ sgn is represented as $x(n)$.
→ For discrete time signal the amplitude b/w 2 time instants is not defined
→ sgn is represented only at discrete instants of time.

$$x(n) = \{ \ldots 0, 0, 1, \underset{\uparrow}{2}, 0, -1, 1, 2 \}$$

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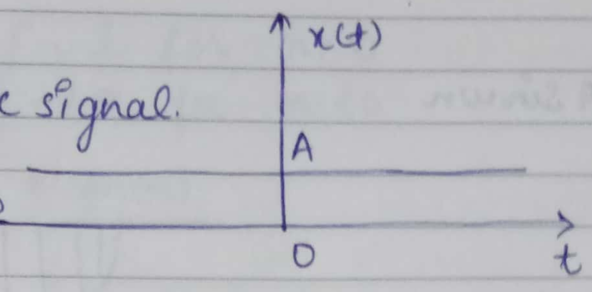
NOTES

ELEMENTRY DISCRETE TIME SEQUENCES.

DC SIGNAL

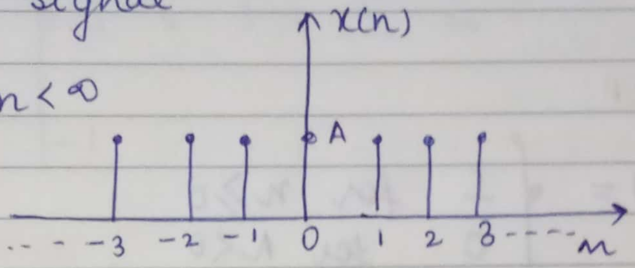
Continuous time DC signal.

$x(t) = A$ for $-\infty < t < \infty$



Discrete time DC signal

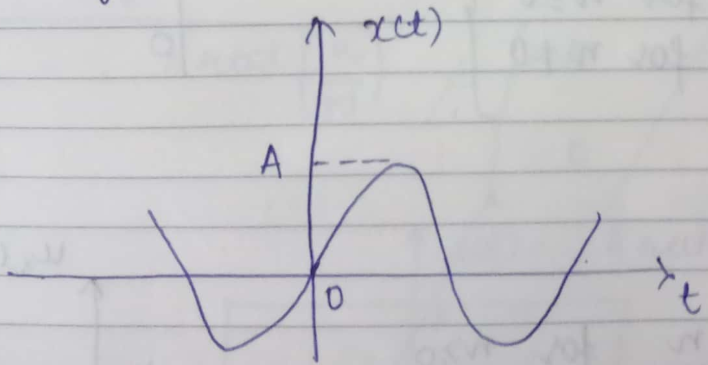
$x(n) = A$ for $-\infty < n < \infty$



SINUSOIDAL SIGNAL

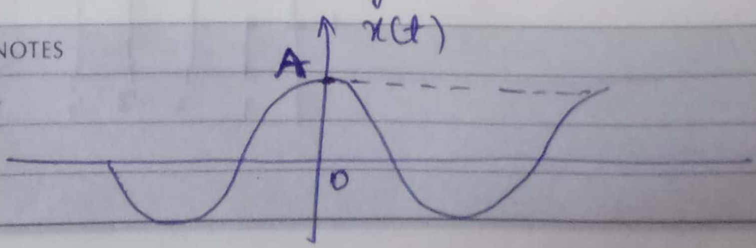
It includes sine and cosine signals

sine signal $x(t) = A \sin \omega t = A \sin(2\pi f t)$

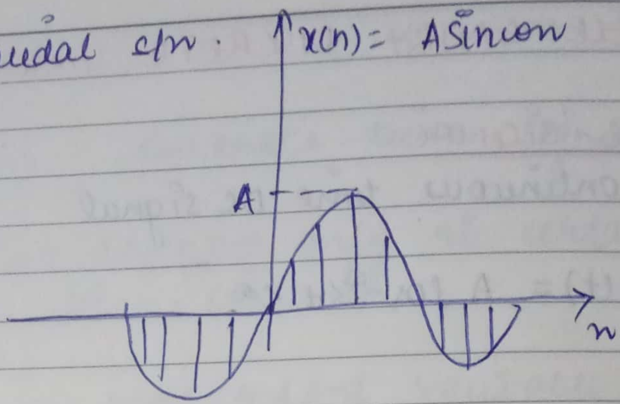


cosine signal $x(t) = A \cos \omega t = A \cos(2\pi f t)$

NOTES



Discrete time sinusoidal s/n.



sine

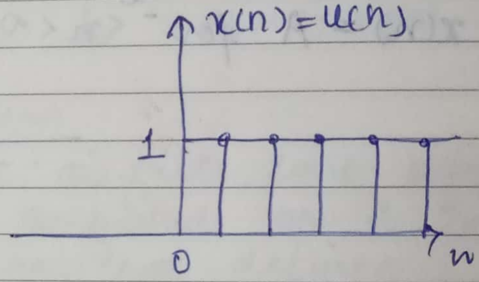
$x(n) = A \sin \omega n$

Illy cosine

$x(n) = A \cos \omega n$

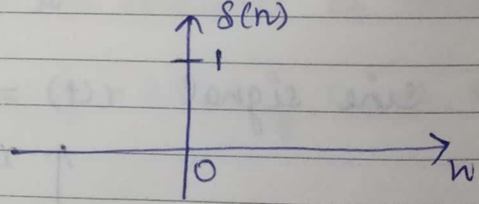
(3) UNIT - STEP

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



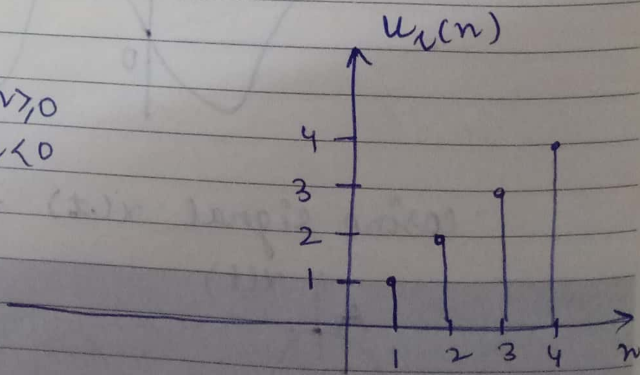
(4) UNIT IMPULSE

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$



(5) UNIT RAMP:-

$$u_r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



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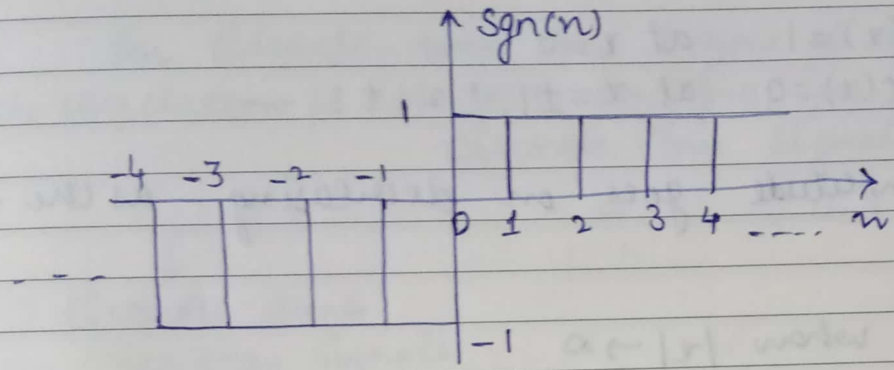
NOTES

(6) SIGNUM FUNCTION

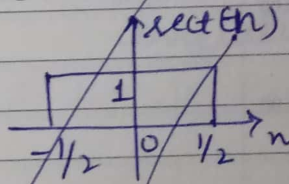
$$\mathrm{sgn}(n) = \begin{cases} 1 & \text{for } n>0 \\ -1 & \text{for } n<0 \end{cases}$$



(7) RECTANGULAR PULSE.

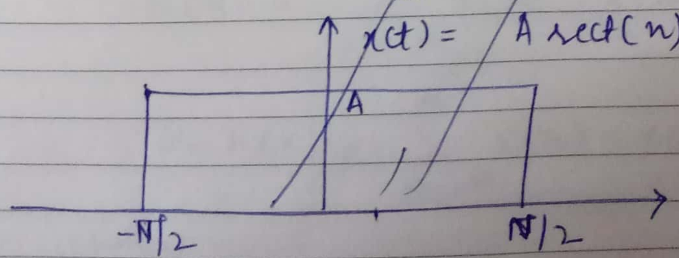
1. Rectangular pulse for unit amplitude and unit duration

$$\mathrm{rect}(n) = \begin{cases} 1 & \text{for } -\frac{1}{2} \leq n \leq \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$



2. Rectangular pulse for Amplitude A and duration T

$$A\,\mathrm{rect}\left(\frac{n}{N}\right) = \begin{cases} A & \text{for } -\frac{N}{2} \leq n \leq \frac{N}{2} \\ 0 & \text{otherwise} \end{cases}$$



NOTES

Sinc Function

$$\text{sinc}(x) = \frac{\sin(\pi x)}{(\pi x)} \quad \text{for } x \neq 0$$

$\text{sinc}(x) = 1$ at $x = 0$

and $\text{sinc}(x) = 0$ at $x = \pm 1, \pm 2, \pm 3$ --

Its value of amplitude goes on decreasing as the value of $|x|$ increases.

$\text{sinc}\, x \to 0$ when $|x| \to \infty$

CORRELATION OF DISCRETE TIME SIGNALS

correlation b/w discrete time signals is the logical extention of correlation of contineous time signals.

For Discrete time s/n 2 types of correlation are there.
Each correlation is applicable for 2 DT sns ① DT Energy sns ② DT Power sns.

- Discrete Time Signals
  - Discrete time energy signals
    - Auto correlation
    - cross correlation
  - Discrete-time power signals.
    - Auto correlation
    - cross correlation

AUTO CORRELATION OF DT ENERGY SIGNALS

Auto correlation is the measure of similarity b/w a sequence $x(n)$ and its shifted version $x(n-K)$

It is given by

$$R(K) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-K)$$

$$\text{or } R(K) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n+K)$$

where

n = variable

K = constant Representing the shift.

AUTO CORRELATION OF DT POWER SIGNALS

Auto correlation f=n of a real valued discrete time power s/n $x(n)$ is defined as:-

$$R(K) = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)\, x(n-K)$$

$$\text{or } R(K) = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)\, x(n+K)$$

CROSS CORRELATION OF D.Time Energy Signals.

Cross correlation is the measure of similarity b/w them.

Let $x_1(n)$ and $x_2(n)$ = pair of real valued discrete-time energy s/ns.

Then cross correlation of such a pair is :-

$$R_{12}(K) = \sum_{n=-\infty}^{N} x_1(n) \cdot x_2(n-K)$$ ✓

$$\text{or } R_{21}(K) = \sum_{n=-\infty}^{\infty} x_1(n-K) \cdot x_2(n)$$

CROSS-CORRELATION OF DISCRETE TIME POWER SIGNALS:-

If $x_1(n)$ and $x_2(n)$ are two different discrete time power signals then the cross-correlation b/w them is def. as

$$R_{12}(K) = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x_1(n)\, x_2(n-K)$$

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NOTES

Then second cross-correlation is defined as

$$R_{21}(K) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x_1(n+k) \cdot x_2(n)$$

- cross correlation f=n is generally non-commulative.
- s/ns $x_1(n)$ and $x_2(n)$ are said to be orthogonal if cross-correlation b/w them is zero i.e for orthogonal signals.

$$R_{12}(K) = 0$$

- If correlation f=n has non-zero value only for -ve value then s/n is called -vely correlated.
- If correlation f=n has non-zero value only for positive value then s/n is called positively correlated.
- If correlation f=n is zero for all the values of τ or k then the signals are said to be uncorrelated s/ns.

EXAMPLE :- Determine the autocorrelation for the sequence

$$x(n) = \{0, 1, 2, \underset{\uparrow}{3}\}$$

SOLUTION :-

Sequence x(n) is discrete time energy signal.

Autocorrelation is def. as

$$R(K) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-k)$$

For $K = (-4)$

$$\therefore R(-4) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n+4)$$

NOTES

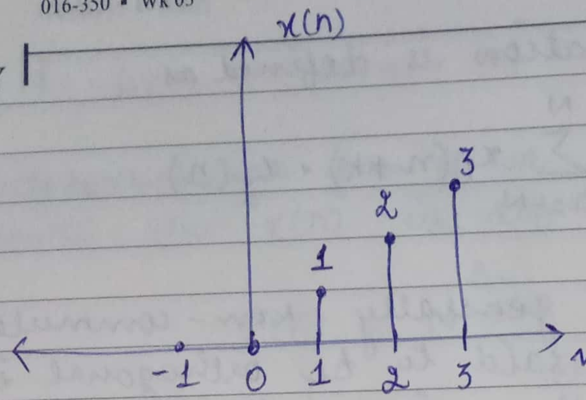
$x(n+4)$ = advance version of $x(n)$

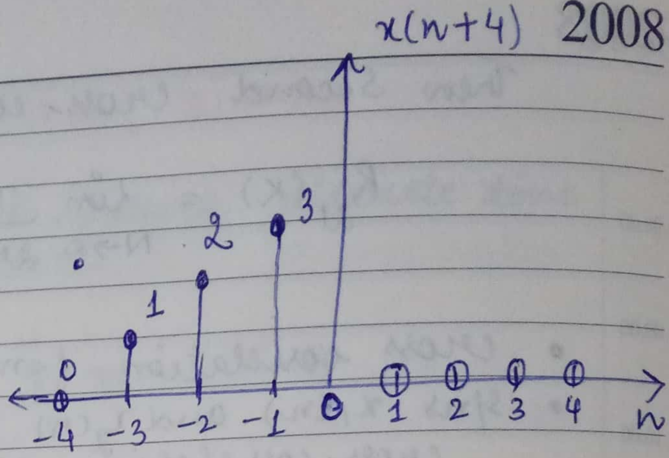
$$R(-4) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n+4)$$

$$\boxed{R(-4) = 0}$$

For $k=-3$ $\quad R(-3) = \sum_{n=-\infty} x(\ldots)$

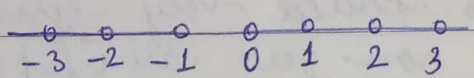
$$\boxed{R(-3) = 0}$$





$x(n) \cdot x(n+4)$

$\Downarrow$



For $K=-2$

$$R(-2) = \sum_{n=-\infty}^{\infty} x(n)\, x(n+2)$$

$$R(-2) = 0+0+0+3+0+0$$

$$\boxed{R(-2) = 3}$$

For $K=-1$

$$R(-1) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-1)$$

$$= x(0)x(1) + x(1)x(2) + x(2)x(3) + \ldots$$

$$= (0\times 1) + (1\times 2) + (2\times 3) + (3\times 0)$$

$$R(-1) = 0+2+6$$

$$\boxed{R(-1) = 8}$$

For $K=0$

$$R(0) = \sum_{n=-\infty}^{\infty} x(n)\cdot x(n)$$

$$= x(0)\,x(0) + x(1)\,x(1) + x(2)\,x(2) + x(3)\cdot x(3) + \ldots$$

$$= (0\times 0) + (1\times 1) + (2\times 2) + (3\times 3)$$

$$= 0+1+4+9$$

$$= 14$$

$$\boxed{R(0) = 14}$$

For k = 1

$$R(1) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-1)$$

$$= x(0)\,x(-1) + x(1)\,x(0) + x(2)\,x(1) + x(3)\,x(2) + x(4)\,x(3)$$
$$= 0 + (1 \times 0) + (2 \times 1) + (3 \times 2) + (0 \times 3)$$
$$= 0 + 0 + 2 + 6 + 0$$

$$\boxed{R(1) = 8}$$

FOR k = 2

$$R(2) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-2)$$

$$= x(0)\,x(-2) + x(1)\,x(-1) + x(2) \cdot x(0) + x(3)\,x(1)$$
$$= (0 \times 0) + (1 \times 0) + (2 \times 0) + (3 \times 1)$$

$$= 0 + 0 + 0 + 3$$

$$\boxed{R(2) = 3}$$

FOR k = 3

$$R(3) = \sum_{n=-\infty}^{\infty} x(n)\,x(n-3)$$

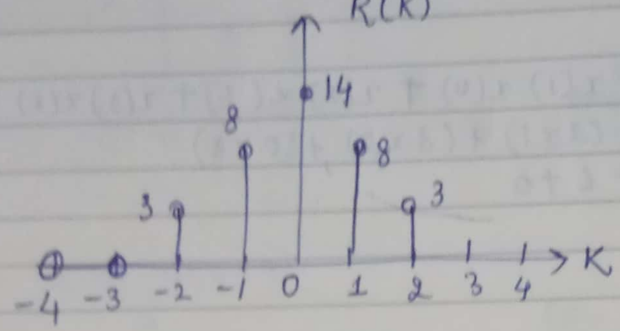
$$= x(0) \cdot x(-3) + x(1)\,x(-2) + x(2)\,x(-1) + x(3)\,x(0)$$
$$= (0 \times 0) + (1 \times 0) + (2 \times 0) + (3 \times 0)$$
$$= 0$$

NOTES

$$\boxed{R(3) = 0}$$

For all values of $K \geq 4$, value of $R(K)$ will be zero.

so auto correlation f=n is plotted like.



- Auto correlation f=n $R(K)$ is symmetrical f=n

i.e $R(K) = R(-K)$

EXAMPLE :- Find the auto correlation of the following s/ns

① $x_1(n) = (4, 3, 2, 1)$

② $x_2(n) = u(n)$

SOLUTION :- ① $x_1(n) = (4, 3, 2, 1)$

Auto correlation

$$R(K) = \sum_{n=-\infty}^{\infty} x_1(n)\, x_1(n-k)$$

- For $k = \pm 4$ there will not be overlap b/w $x_1(n)$ and $x_2(n-4)$.

Hence the product will be zero

So we have to find $R(k)$ in the Range $-3 \leq K \leq 3$

Thus

$$R(-3) = \sum_{n=-\infty}^{\infty} x_1(n) \cdot x_1(n+3)$$

$$R(-3) = x_1(-1)\,x_1(2) + x_1(0)\,x_1(3) + x_1(1) \cdot x_1(4) + x_1(2) \cdot x_1(5)$$

$$R(-3) = (4 \times 1) + (3 \times 0) + (2 \times 0) + (1 \times 0)$$

$$\boxed{R(-3) = 4}$$

For $K = -2$

$$R(-2) = \sum_{n=-\infty}^{\infty} x_1(n)\, x_2(n+2)$$

$$= \sum_{n=-2}^{2} x_1(n) \cdot x_1(n+2)$$

$$= x_1(-2) \cdot x_1(0) + x_1(-1) \cdot x_1(1) + x_1(0)\,x_1(2) + x_1(1) \cdot x_1(3) + x_1(2) \cdot x_1(4)$$

$$= (0 \times 4) + (0 \times 3) + (4 \times 2) + (3 \times 1) + (2 \times 0)$$

$$= 0 + 0 + 8 + 3 + 0$$

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$$\boxed{R(-2) = 11}$$

IIly $\boxed{R(-1) = 20}$

$$\boxed{R(0) = 30}$$

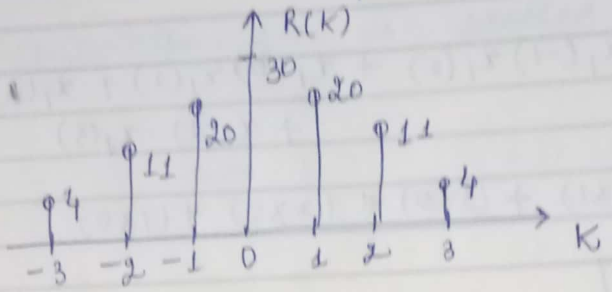
NOTES

$$\boxed{R(1) = 20}$$

$$\boxed{R(2) = 11}$$

$$\boxed{R(3) = 4}$$

$R(K) = (4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4)$



(2) $x_2(n) = u(n)$

auto-correlation $R(K) = \sum_{n=-\infty}^{\infty} x_2(n)\, x_2(n-k)$

For $u(n)$

$$R(K) = \sum_{n=\infty}^{\infty} u(n)\, u(n-1)$$

But $u(n) \neq 0$ for $n \geq 0$
Hence limits of summation will be from 0 to ∞

For $K = -\infty$ to ∞ even though the overlapping changes, there will not be any change in $R(K)$ because $u(n)$ and $u(n-K)$ both are infinite sequences

$\therefore$ $R(K)$ will be constant but will not have a finite value.

EXAMPLE

obtain the cross-correlation of the following sequences

$$x_1(n) = [2, 3, 4] \quad (\uparrow \text{ at } 2)$$

$$x_2(n) = [1, 2, 3] \quad (\uparrow \text{ at } 2)$$

SOLUTION :- we solve this by 2 methods

① DIRECT COMPUTATION METHOD
② GRAPHICAL METHOD

DIRECT COMPUTATION METHOD

cross-correlation is def. as

$$R_{12}(K) = \sum_{n=-\infty}^{\infty} x_1(n) \cdot x_2(n-K)$$

Let us find out $R_{12}(K)$ for different values of K.

Since $x_1(n)$ exist from $n=0$ to 2 the product term $x_1(n) x_2(n-K)$ will be non-zero only for these value of n.

$$\therefore R_{12}(K) = \sum_{n=0}^{2} x_1(n) \cdot x_2(n-K)$$

① $R_{12}(K)$ for $K = -3$

$$R_{12}(-3) = \sum_{n=0}^{2} x_1(n) \cdot x_2(n+3)$$

$$= x_1(0) \cdot x_2(3) + x_1(1) \cdot x_2(4) + x_1(2) \cdot x_2(5)$$
$$= (2 \times 0) + (3 \times 0) + (4 \times 0) = 0$$

$$\boxed{R_{12}(-3) = 0}$$

EXAMPLE

obtain the cross-correlation of the following sequences

$$x_1(n) = [\underset{\uparrow}{2}, 3, 4]$$

$$x_2(n) = [1, \underset{\uparrow}{2}, 3]$$

SOLUTION :- we solve this by 2 methods

1. DIRECT COMPUTATION METHOD
2. GRAPHICAL METHOD

DIRECT COMPUTATION METHOD

cross-correlation is def. as

$$R_{12}(K) = \sum_{n=-\infty}^{\infty} x_1(n) \cdot x_2(n-K)$$

Let us find out $R_{12}(K)$ for different values of K.

Since $x_1(n)$ exist from $n=0$ to 2 the product term $x_1(n)\, x_2(n-K)$ will be non-zero only for these value of n.

$$\therefore R_{12}(K) = \sum_{n=0}^{2} x_1(n) \cdot x_2(n-K)$$

(1) $R_{12}(K)$ for $K = -3$

$$R_{12}(-3) = \sum_{n=0}^{2} x_1(n) \cdot x_2(n+3)$$

$$= x_1(0) \cdot x_2(3) + x_1(1) \cdot x_2(4) + x_1(2) \cdot x_2(5)$$
$$= (2\times 0) + (3\times 0) + (4\times 0) = 0$$

$$\boxed{R_{12}(-3) = 0}$$

(2) $R_{12}(K)$ For $K=-2$

$$R_{12}(-2) = \sum_{n=0}^{2} x_1(n) \cdot x_2(n+2)$$

$$= x_1(0) \cdot x_2(2) + x_1(1)\, x_2(3) + x_1(2)\, x_2(4)$$

$$= (2 \times 3) + (3 \times 0) + (4 \times 0)$$

$$\boxed{R_{12}(-2) = 6}$$

0

(3) $R_{12}(K)$ for $K=-1$

$$R_{12}(-1) = \sum_{n=0}^{2} x_1(n) \cdot x_2(n+1)$$

$$= x_1(0) \cdot x_2(1) + x_1(1)\, x_2(2) + x_1(2)\, x_2(3)$$

$$= (2 \times 2) + (3 \times 3) + (4 \times 0)$$

$$= 4 + 9 + 0$$

$$\boxed{R_{12}(-1) = 13}$$

6

(4) $R_{12}(K)$ for $K=0$

$$R_{12}(0) = \sum_{n=0}^{2} x_1(n)\, x_2(n)$$

$$= x_1(0)\, x_2(0) + x_1(1)\, x_2(1) + x_1(2)\, x_2(2)$$

$$= (2 \times 1) + (3 \times 2) + (4 \times 3)$$

$$\boxed{R_{12}(0) = 20}$$

13

(5) $R_{12}(K)$ for $k=1$

$$\boxed{R_{12}(1) = 11}$$

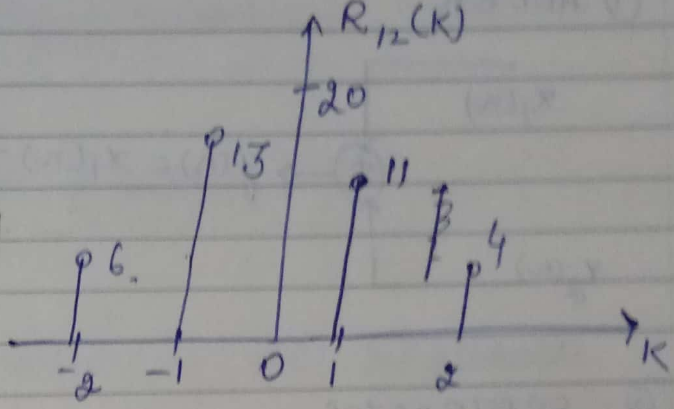
20

⑥ $R_{12}(K)$ for $K=2$.

$$\boxed{R_{12}(2) = 4}$$

⑦ $R_{12}(K)$ for $K=3$

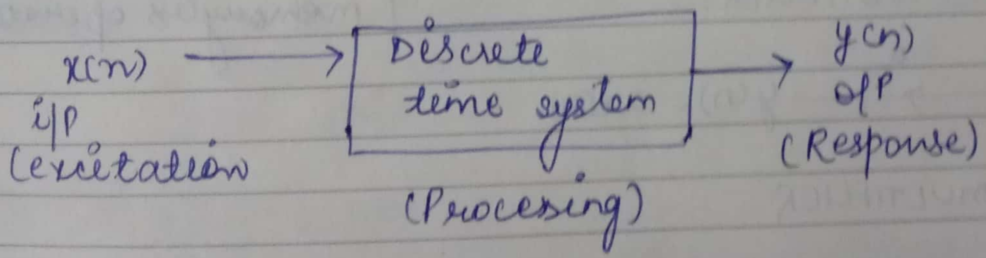
$$\boxed{R_{12}(3) = 0}$$



$$R_{12}(K) = (6, 13, \underset{\uparrow}{20}, 11, 4)$$

DISCRETE TIME SYSTEMS

It is a physical device which performs req. operation on a discrete time signal.



when i/p s/n is passed through the system then it is represented as

$$x(n) \xrightarrow{T} y(n)$$

NOTES

$$y(n) = T\ x(n)$$

T = T'formation operation.

eg = filter

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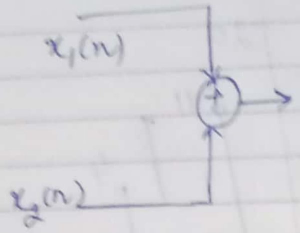
SYMBOLS TO REPRESENT DT SYSTEM

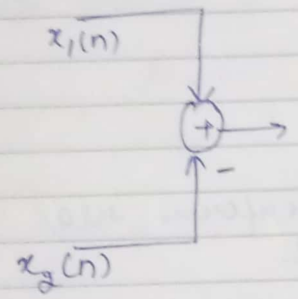
(1) ADDER



$y(n) = x_1(n) + x_2(n)$

[memoryless operation]

(2) SUBTRACTOR



$y(n) = x_1(n) + (-x_2(n))$

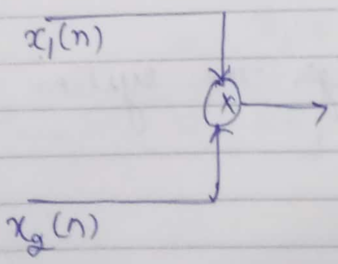
$y(n) = x_1(n) - x_2(n)$

(3) CONSTANT MULTIPLIER.

[memoryless operation]

$x(n) \xrightarrow{a} y(n) = a x(n)$

(3) SIGNAL MULTIPLIER.



$y(n) = x_1(n) \cdot x_2(n)$

[memoryless operation]

(4) UNIT DELAY. [Requires memory to store previous input]

$$x(n) \longrightarrow \boxed{z^{-1}} \longrightarrow y(n) = x(n-1)$$

$$x(n) \longrightarrow \boxed{z^{-1}} \xrightarrow{x(n-1)} \boxed{z^{-1}} \longrightarrow y(n) = x(n-2)$$

(5) UNIT ADVANCE [This operation is not realizable in practice]

$$x(n) \longrightarrow \boxed{z} \longrightarrow y(n) = x(n+1)$$

CLASSIFICATION OF DISCRETE TIME SYSTEMS.

(1) DYNAMIC AND STATIC

STATIC :- It is a system in which output at any instant of time depends upon input sample at the same time.

eg:- (1) $y(n) = 5\,x(n)$

5 is a constant which multiplies i/p $x(n)$.

But ~~Both~~ output at n^{th} instant, means $y(n)$ depends on i/p at the same time (n^{th}) time instant of $x(n)$.

Hence it is static system.

(2) $y(n) = x^2(n) + 5x(n) + 10$

NOTES

DYNAMIC:- It is a system in which output at any instant of time depends on input sample at the same time as well as at other times means past and future time.

$x(n)$ = original s/n.
$x(n-k)$ = delayed s/n
$x(n+k)$ = advanced signal

eq:- (1) $y(n) = x(n) + 5x(n-1)$

(2) $y(n) = 3x(n+2) + x(n)$.

Example:-

(1) $y(n) = n\,x(n)$
static

(2) $y(n) = x(n^2)$
Dynamic becoz at n^{th} instant, system expects future inputs n^2.

for eg at $n=1$; $y(n) = x(1^2) = x(1)$
at $n=2$; $y(n) = x(2^2) = x(4)$.

at $n=2$, system req. future i/p $x(4)$.

(3) $y(n) = x(n) - 3\,x(n-2)$
Dynamic

(4) $y(n) = x(-n)$
dynamic. put diff. values of n.

$n=0$; $y(n) = x(0)$
$n=1$; $y(n) = x(-1)$
$n=2$; $y(n) = x(-2)$

$x(-1), x(-2)$ = past i/p signals. Hence dynamic.

TIME VARIENT AND INVARIENT

TIME INVARIENT :- Time invarient is if its i/p-o/p characteristics do not change with time

$x(n) \longrightarrow \boxed{\text{System } (T)} \longrightarrow y(n)$ $\quad x(n-K) \longrightarrow \boxed{\text{System } T} \longrightarrow y(n-K)$

$x(n) \xrightarrow{T} y(n)$

$x(n-K) \xrightarrow{T} y(n-K)$

Time varient :- whose i/p-o/p characteristics change with time.

eg:- Determine whether the system is Time invarient or not

$$y(n) = x(n) - x(n-1)$$

Soln:- ① 1st delay the i/p by k samples and denote o/p by $x(n,k)$

$$y(n,K) = x(n-K) - x(n-K-1)$$

② Replace n by n-K throughout the eqn.

$$y(n-K) = x(n-K) - x(n-K-1)$$

compare ① and ② both are same.
Hence it is TIV system.

NOTES

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(3) linear and non linear system

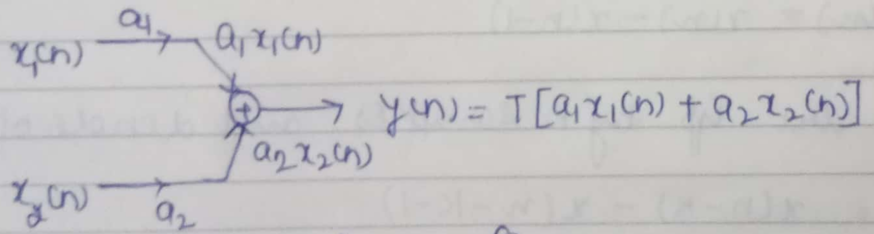
LINEAR:- It is a system which follows superposition principle.

$$x(n) \rightarrow \boxed{\text{System } T} \rightarrow y(n) = Tx(n)$$

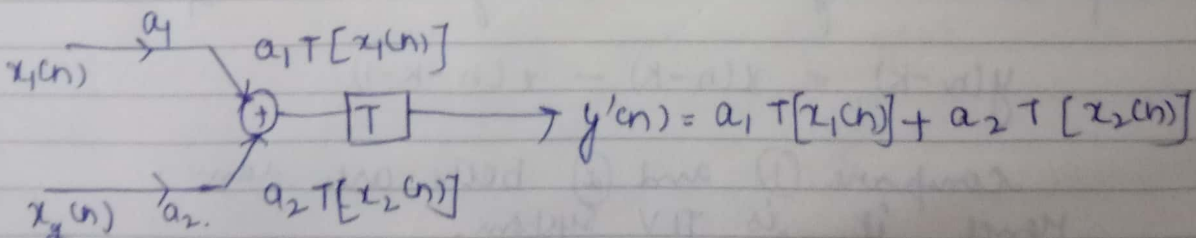
Let $x_1(n)$ produces $y_1(n)$
and $x_2(n)$ produces $y_2(n)$
consider there are 2 arbitrary constants a_1 and a_2.
Now
$a_1 x_1(n)$ produces output $a_1 y_1(n)$
$a_2 x_2(n)$ produces output $a_2 y_2(n)$.

Theorem:-

$$T[a_1x_1(n) + a_2x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)] \quad \text{— (A)}$$



L.H.S of eq^n (A)



NOTES:- If i/p = 0 then o/P = 0
but if it produces some o/P
it becomes non-linear system.

Example: Determine whether the system is linear or not.

$$y(n) = n x(n)$$

(1) when i/p $=0$; o/p $=0$

If $x(n) = 0$

$y(n) = n(0)$

$y(n) = 0.$

system is linear.

(2) If we have 2 i/ps then.

$x_1(n) \xrightarrow{T} y_1(n) = n x_1(n)$

$x_2(n) \xrightarrow{T} y_2(n) = n x_2(n)$

add these (2)

$y'(n) = y_1(n) + y_2(n) = n[x_1(n) + x_2(n)]$ — (1)

or $x_1(n) + x_2(n) \xrightarrow{T} n[x_1(n) + x_2(n)]$

we know that f=n of system is to be multplied i/p by (n)

Hence $[x_1(n) + x_2(n)]$ = one i/p to the system.

o/p will be

$y''(n) = n[x_1(n) + x_2(n)]$ — (2)

Compare (1) and (2)

$y'(n) = y''(n)$

Hence system is linear

NOTES

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Causal not causal

Causal :- system is said to be causal if output at any instant of time depends only on present and past inputs. But o/p does not depend upon future i/ps.

eg:- $y(n) = x(n) + x(n-1)$

$y(n) = x(n) + x(n-1)$

$y(n) = 3x(n)$.

- system is practically realizable.
- all real time systems are causal.
- memoryless system

Non causal

system is anti causal if its output depends not only on present and past but also on future i/ps.

eg:- $y(n) = x(n) + x(n+1)$

$y(n) = x(n+2)$

$y(n) = x(n) + n x(n+4)$

- practically not realizable
- if sgns are stored in a memory and later they are used by system such sgns are treated as future signals.

STABLE:- An initially relaxed system is BIBO stable if and only if every bounded i/p produces bounded output.

Relaxed system means when i/p of system is zero then output of the system is also zero.

i/p is bounded

$|x(n)| < m_x < \infty$

o/p is bounded

$|y(n)| < m_x < \infty$

where m_x = some finite number.

UNSTABLE:- An initially relaxed system is said to be unstable if bounded i/p produces unbounded output.

→ these systems are erratic and extreme behaviour

→ when unstable system is practically implemented then it causes overflow.

Examples:- ① $y(n) = \cos[x(n)]$

for every bounded value of x(n), its cosine value is always bounded.

So given system is stable

SUNDAY 3

② $y(n) = x(-n)$

NOTES

$x(-n)$ = folding of i/p sequence

- means taking mirror image of x(n).
- in folding operation amp. of Sn is not changed. So if i/p is Bounded $y(n) = x(-n)$ will be bounded.

(stable system)

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CONVOLUTION

multipiication of 2 s/ns

→ One is stationary and one is moving

$h(n)$ = empulse function, It is a characteristic of a system How i/p behaves when it passes through a system (machine will tell this)

Four methods of convolution

① Graphical method

② using mathamatical eq=n of convolution

③ Tabulation method

④ multiplication method.

① GRAPHICAL METHOD.

Linear convolution of 2 sequences is given by

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$

Example:- obtain linear convolution of following sequences.

$x(n) = \{1, 2, 1, 2\}$ and $h(n) = \{1, 1, 1\}$.

Soln:- If the sequences are given in terms of n then we obtain $X(k)$ and $h(k)$ by replacing n by K.

So.

$x(k) = \{1, 2, 1, 2\}$

$h(k) = \{1, 1, 1\}$

$$\therefore \quad y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k). \quad -(1)$$

Now decide the value or range of y(n) and k.

To calculate the range of y(n) or n

$y_1 = x_1 + h_1$ and $y_2 = x_2 + h_2$.

$x_1 = 0$, $x_2 = 3$

$h_1 = 0$, $h_2 = 2$

$y_1 = x_1 + h_1 = 0 + 0 = 0$

$y_2 = 3 + 2 = 5$

Thus range of y(n) is 0 to 5

and k is from 0 to 3
[the range of x(k) only]

k ranges from 0 to 3 then eq=n (1) becomes

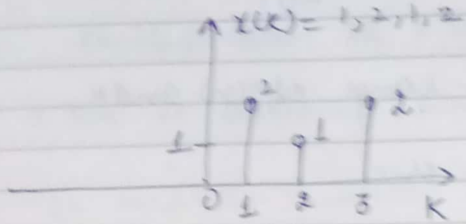
$$y(n) = \sum_{k=0}^{3} x(k)\, h(n-k) \quad -(2)$$

(i) Calculation of y(0)

$$y(0) = \sum_{k=0}^{3} x(k)\, h(0-k)$$

$$y(0) = \sum_{k=0}^{3} x(k)\, h(-k) \quad -(3)$$

Sketch $x(k)$



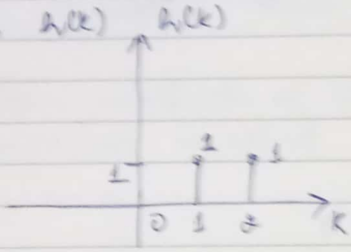
08.00

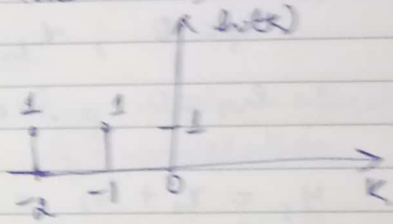
09.00

10.00

Sketch $h(k)$

Fold $h(k)$ to obtain $h(-k)$





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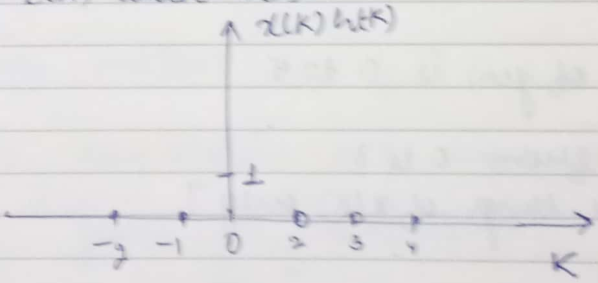
01.00

02.00

Multiply $x(k)$ with $h(-k)$



03.00

04.00

05.00

06.00

the summation of all product terms

$$\boxed{y(0) = 1}$$

② Calculation of $y(1)$; Put $n=1$

$$y(1) = \sum_{k=0}^{3} x(k)\, h(1-k)$$

it can also be written as

$$y(1) = \sum_{k=0}^{3} x(k)\, h(-k+1)$$

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24	25	26	27	28	29	

NOTES

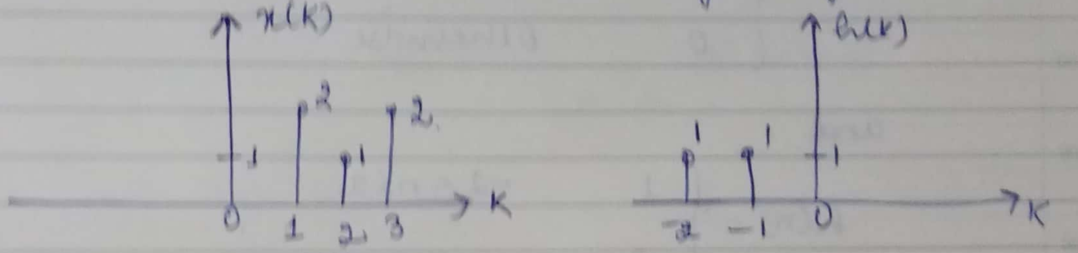
$h(-k+1)$

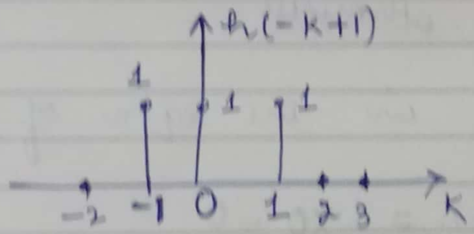
$h(k-1)$

$h(n-k)$

$h(-k+1)$ is delay of $h(-k)$ by 1 samples

Hence shift the sequence towards right by 1 sample.



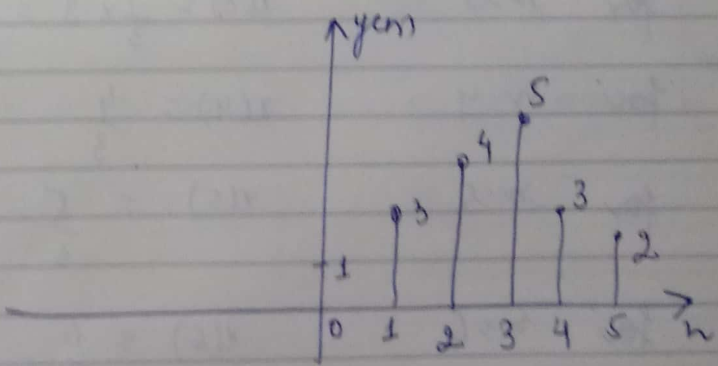


$y(1) = (1 \times 1) + (1 \times 2) + 0 + 0$

$= 1 + 2$

$= 3$

$\boxed{y(1) = 3}$



11ly $\boxed{y(2) = 4}$

$\boxed{y(3) = 5}$

$\boxed{y(4) = 3}$

NOTES

$\boxed{y(5) = 2}$

$\boxed{y(n) = 1, 3, 4, 5, 3, 2}$

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TABULAR FORM

Que:- Compute the convolution $y(n)$ of the following signals

$$x(n) = \begin{cases} \frac{1}{3}n & 0 \le n \le 6 \\ 0 & \text{Otherwise} \end{cases}$$

and

$$h(n) = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Solution put $n = 0$ to 6 in the eq=n of $x(n)$

for $n=0$ $\quad x(0) = \frac{1}{3}n = \frac{1}{3} \times 0 = 0$

for $n=1$ $\quad x(1) = \frac{1}{3} \times 1 = \frac{1}{3}$

for $n=2$ $\quad x(2) = \frac{1}{3} \times 2 = \frac{2}{3}$

for $n=3$ $\quad x(3) = \frac{1}{3} \times 3 = \frac{3}{3} = 1$

for $n=4$ $\quad x(4) = \frac{4}{3}$

for $n=5$ $\quad x(5) = \frac{5}{3}$

for $n=6$ $\quad x(6) = \frac{6}{3} = 2$

$\therefore$ $x(n) = \left\{0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\right\}$

$x(k) = \left\{0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\right\}$

Ily $n = -2$ to n for $h(n)$

$h(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

8.00

9.00

Acc. to eq=n of convolution

10.00

$$y(n) = x(n) * h(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$

11.00

Range of K = 0 to 6

Range of n = -2 to 8

12.00

$$y(n) = \sum_{K=0}^{6} x(K) . h(n-K)$$

1.00

Tabular method.

2.00

	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2

3.00

4.00

5.00

6.00

SUNDAY 10

$$y(n) = \left\{0, \frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\right\}$$

NOTES

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(3) USING MATHAMATICAL EQUATION OF CONVOLUTION

simply by opening the submission and get value added to find $y(n)$

08.00

09.00

(4) MULTIPLICATION METHOD

10.00

$x(n) = \{1 \quad 2 \quad 1 \quad 2\}$

11.00

$h(n) = \{1, 1, 1\}$

12.00

$$
\begin{array}{r}
1 \quad 2 \quad 1 \quad 2 \\
\times \quad 1 \quad 1 \quad 1 \\
\hline
1 \quad 2 \quad 1 \quad 2 \\
1 \quad 2 \quad 1 \quad 2 \quad \times \\
1 \quad 2 \quad 1 \quad 2 \quad \times \quad \times \\
\hline
\end{array}
$$

01.00

02.00

03.00

$\{1, 3, 4, 5, 3, 2\}$

04.00

LINEAR CONSTANT COEFFICIENT DIFFERENCE EQUATION

05.00

O/P of discrete time system can be calculated by linear convolution

06.00

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

$h(n) =$ impulse Response of the system

→ Difference eq^n is useful to implement the discrete time system
→ specially used for designing the digital filter.

$h(n)$ is used to calculate $y(n)$ of DT system.

If the system is causal
$h(n) = 0$ for all the -ve values of n.

∴ Causal LTI system eq^n is

$$y(n) = \sum_{k=0}^{\infty} x(k) \cdot h(n-k)$$

or

$$y(n) = \sum_{k=0}^{\infty} h(k) \cdot x(n-k)$$

→ $y(n)$ depends upon $h(n)$
→ No. of samples present in $h(n)$ is called as length of impulse Response

Depending upon length of $h(n)$, there are 2 types of discrete time systems

1. FIR systems
2. IIR systems

FIR System:-

As name indicates $h(n)$ is having finite length.
→ $h(n)$ contains finite no. of samples.

Let Range of $h(n)$ is from $-M$ to M.

∴ eq:n of convolution for FIR system :-

$$y(n) = \sum_{K=-M}^{M} x(K) \cdot h(n-K) \qquad \text{(Non- causal)}$$

For causal system we consider only +ve terms

$$\therefore \quad y(n) = \sum_{K=0}^{M} x(K) \cdot h(n-K) \qquad \text{(Causal)}$$

IIR SYSTEM:-

$$y(n) = \sum_{K=-\infty}^{\infty} h(K) \cdot x(n-K) \qquad \text{Non- Causal.}$$

$$y(n) = \sum_{K=0}^{\infty} h(K) \cdot x(n-K) \qquad \text{Causal.}$$

Depending upon o/p of the system, DT system are of 2 types:
(1) Non-Recursive systems
(2) Recursive systems.

NOTES

Non-Recursive :-
- do not require any past o/p samples to calculate present o/p.
- no feedback.

Recursive Systems

- with feedback
- depends upon present, past inputs.

LTI SYSTEM CHARACTERIZED BY CONSTANT - COEFFICENT DIFFERENCE EQUATION

Behaviour of system depends on its impulse Response.

→ Now in this we discuss how LTI systems are characterized by i/p-o/p relations called as diff. eq-ns with constant co-efficients.

Let us consider Recursive system
eq=n of o/P :-

$$y(n) = x(n) + a\,y(n-1) \quad -(1)$$

This is first order eq=n.
a = constant co-eff.

$y(n)$ is calculated by putting diff value of n in eq=n (1)

For $n=0$; $y(0) = x(0) + a\,y(-1)$ — (2)
for $n=1$; $y(1) = x(1) + a\,y(0)$ — (3)

Put the value of $y(0)$ from eq=n (2) to eq=n (3)

$$y(1) = x(1) + a[x(0) + a\,y(-1)]$$

$$y(1) = x(1) + a\,x(0) + a^2 y(-1)$$

NOTES

Rearranging the terms.

$$y(1) = a^2 y(-1) + a\,x(0) + x(1) \quad -(4)$$

||ly for n values $y(n) = a^{n+1} y(-1) + \sum_{k=0}^{n} a^k x(n-k)$ — (A)

1) → $y(-1)$ = Initial condition of system.

2) → 2^{nd} term = response of the system to i/p $x(n)$.

depending upon these 2 initial conditions, 2 types of Responses are obtained

① ZERO STATE OR FORCED RESPONSE (Particular)

If $y(-1) = 0 \Rightarrow$ Zero state Response.

→ o/p is becoz of $x(n)$ only
→ since system is forced to the i/p so called forced Response
→ since $y(-1) = 0$ so it is also called as Relaxed system.

Represented by

$$y_{zs}(n) = \sum_{k=0}^{n} a^k . x(n-k) \quad \text{for } n > 0.$$

② ZERO INPUT RESPONSE (Homogeneous) or natural Response

$y(n) \neq 1$
and $x(n) = 0$

[i/p is not applied to the system and $y(-1) \neq 1$]

→ non-Relaxed system.
Represented by

$$y_{zi}(n) = a^{n+1} y(-1) \quad \text{for } n \geq 0$$

o/p is not dependent on i/p $x(n)$ but depends on $y(-1)$

$$y(n) = b_0 x(n) + b_1 x(n-1) + \cdots b_M x(n-M) - a_1 y(n-1) - \cdots a_N y(n-N)$$

$$= \sum_{K=0}^{M} b_K x(n-K) - \sum_{K=1}^{N} a_K y(n-K)$$

③ TOTAL RESPONSE

$$\boxed{y(n) = y_{zi}(n) + y_{zs}(n)}$$

$$y(n) = \underbrace{-\sum_{K=1}^{N} a_K \cdot y(n-K)} + \sum_{K=0}^{M} b_K \cdot x(n-K)$$

SOLUTION OF LINEAR CONSTANT CO-EFF. DIFFERENCE EQUATION

$$y(n) = y_p(n) + y_h(n)$$

Homogeneous solⁿ of a diff. eq=n becomes

$$\sum_{K=0}^{N} a_K\, y(n-K) = 0 \quad \text{—(A)}$$

We assume solⁿ is in the form of exp

$$y_h(n) = \lambda^n$$

∴ eq=n (A) becomes

$$\sum_{K=0}^{N} a_K\, \lambda^{n-K} = 0$$

open it and take lowest limit outside

$$a\,\lambda^{n-N}\left(\lambda^N + a_1\lambda^{N-1} + a_2\lambda^{N-2} + \cdots a_{N-1}\lambda + a_N\right) = 0$$

find out roots. $\lambda_1, \lambda_2 \cdots \lambda_N$

general solution

$$y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n + \cdots C_N\lambda^N$$

NOTES

$C_1, C_2 \cdots C_N$ weighted co-eff.

For Homogeneous solⁿ $x(n) = 0$

Que1. Determine homogeneous solⁿ for diff. eq=n

$$y(n) - 3y(n-1) - 4y(n-2) = 0 \quad \text{—} (1)$$

Solⁿ Ist Put $y_h(n) = \lambda^n$

Put in eq=n (1)

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}[\lambda^2 - 3\lambda - 4] = 0$$

find Roots

$\boxed{\lambda = -1 \text{ and } 4}$ $\quad \left[\begin{matrix}\lambda_1 = -1 \\ \lambda_2 = 4\end{matrix}\right]$

General ~~solⁿ to~~ form of homogeneous eq=n.

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n \quad \text{—} (2)$$

Put the values of λ_1 and λ_2 in eq=n (2)

$$y_h(n) = C_1(-1)^n + C_2(4)^n \quad \text{—} (3)$$

Now obtain the values of C_1 and C_2.

Put $n=0$ and 1 ~~for~~ in eq=n (1)

and $n=0$ and 1 in eq=n (3)

we will get 4 eq=ns.

Put $n=0$ in eq=n (1)

~~$y(0) - 3y(0) - 4y(-1) = 0$~~

~~or $y(0) = 3y(0) + 4y(-1)$~~

Note:-

$y(0) - 3y(-1) - 4y(-2) = 0$

or $y(0) = 3y(-1) + 4y(-2)$ — (4)

Put $n=1$ in eq=n (1)

$y(1) - 3y(0) - 4y(-1) = 0$

or $y(1) = 3y(0) + 4y(-1)$ — (5)

Put the value of $y(0)$ from eq=n (4) to eq=n (5)

$y(1) = 3[3y(-1) + 4y(-2)] + 4y(-1)$

$y(1) = 9y(-1) + 12y(-2) + 4y(-1)$

$y(1) = 13y(-1) + 12y(-2)$ —— (7)

Now put $n=0$ in eq=n (3)

$y(0) = c_1 + c_2$ —— (8)

put $n=1$

$y(1) = -c_1 + 4c_2$ — (9)

compare (4) and (8)

$c_1 + c_2 = 3y(-1) + 4y(-2)$ — (A)

compare (5) and (9)

$-c_1 + 4c_2 = 13y(-1) + 12y(-2)$ — (B)

Equate A and B and calculate values for c_1 and c_2

NOTES

$c_1 + c_2 = 3y(-1) + 4y(-2) \quad \times 4$

$-c_1 + 4c_2 = 13y(-1) + 12y(-2) \quad \times 1$

$$4c_1 + 4c_2 = 12y(-1) + 16y(-2)$$
$$-9c_1 + 4c_2 = 13y(-1) + 12y(-2)$$

08.00 $$5c_1 = -y(-1) + 4y(-2)$$

09.00 $$c_1 = -\frac{1}{5}y(-1) + \frac{4}{5}y(-2)$$

10.00 Put value of c_1 in eq=n (A)

11.00 $$-\frac{1}{5}y(-1) + \frac{4}{5}y(-2) + c_2 = 3y(-1) + 4y(-2)$$

12.00

01.00 $$c_2 = 3y(-1) + \frac{1}{5}y(-1) + 4y(-2) - \frac{4}{5}y(-2)$$

02.00 $$c_2 = \frac{16}{5}y(-1) + \frac{16}{5}y(-2).$$

03.00

04.00 Put value of c_1 and c_2 in eq=(2)

05.00 $$y_h(n) = \left[-\frac{1}{5}y(-1) + \frac{4}{5}y(-2)\right](-1)^n + \left[\frac{16}{5}y(-1) + \frac{16}{5}y(-2)\right](4)^n$$

06.00 Que:- ~~$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$~~

Determine homogeneous solⁿ of the system described by 1st order diff. eq=n

$$y(n) + a_1 y(n-1) = x(n) \quad \text{—}(1)$$

solⁿ Put $x(n) = 0$

eq=n becomes

$$y(n) + a_1 y(n-1) = 0$$

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NOTES

Put $y(n) = \lambda^n$ in eq=n ①

$$\lambda^n + a_1 \lambda^{n-1} = 0$$

$$\lambda^{n-1} [\lambda + a_1] = 0$$

$$\boxed{\lambda = -a_1}$$

∴ solution to homogeneous diff. eq=n is

$$y_h(n) = C\lambda^n$$

$$y_h(n) = C(-a_1)^n \quad —②$$

Put $n=0$ in eq=n ①

$$y(0) + a_1(y(-1)) = 0$$

$$y(0) = -a_1 y(-1) \quad —③$$

Put $n=0$ in eq=n ②

$$y_h(0) = C(-a_1)^0$$

$$y_h(0) = C. \quad —④$$

$$\therefore y_h(n) = y(0)(-a_1)^n.$$

zero input response

$$y_{zi}(n) = (-a_1)^{n+1} y(-1).$$

NOTES

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Particular solution to Difference equation.

Que:- Determine particular solution of $|a_1| < 1$

08.00 $y(n) + a_1 y(n-1) = x(n)$

09.00 where i/p $x(n)$ is a unit step sequence

$x(n) = u(n)$

10.00

11.00 **Sol:-** $y_p(n) = K\,u(n)$

12.00 $K\,u(n) + a_1 K\,u(n-1) = u(n)$

01.00 $u(n) = u(n-1) = 1$

02.00 $K + a_1 K = 1$

03.00 $\boxed{K = \dfrac{1}{1+a_1}}$

04.00

$\therefore\ y_p(n) = \dfrac{1}{1+a_1} \cdot u(n)$

05.00

06.00 **Que:-** Determine P.S for

$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$

$x(n) = 2^n$

$n \geq 0.$

$\boxed{y_p(n) = K\,x(n)}$

Sol:- Put $y_p(n) = K 2^n$.

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NOTES

Total solⁿ

Que:- Determine total solution of the difference eq=n

$$y(n) + a_1 y(n-1) = x(n) \quad -(A)$$

where $x(n) = u(n)$

$y(-1)$ is the initial condition.

Solⁿ:-

$$y(n) + a_1 y(n-1) = x(n) \quad -(A)$$

for homogeneous solⁿ.

put $x(n) = 0$

$$y(n) + a_1 y(n-1) = 0$$

Put $y(n) = \lambda^n$

$$\lambda^n + a_1 \lambda^{n-1} = 0$$

$$\lambda^{n-1}[\lambda + a_1] = 0$$

$$\boxed{\lambda = -a_1}$$

$$y_h(n) = C(-a_1)^n.$$

For Particular solⁿ.

Put $y_p(n) = K \cdot u(n)$

$$K \cdot u(n) + a_1 K(u(n-1)) = u(n)$$

$$u(n) = u(n-1) = 1$$

$$K + a_1 K = 1$$

$$K = \frac{1}{1+a_1}$$

$$c = \frac{\cancel{1} + a_1 - \cancel{1}}{1+a_1}$$

$$\boxed{c = \frac{a_1}{1+a_1}}$$

Total sol.

$$\boxed{\therefore y(n) = \frac{a_1}{1+a_1}(-a_1)^n + \frac{1}{1+a_1}}$$

Note

If $y(-1) \neq 0$ then

Put $n=0$ in eq-n (A)

$$y(0) + a_1 y(-1) = 1$$

$$y(0) = 1 - a_1 y(-1) \quad \text{--- (1)}$$

Put $n=0$ in eq-n (C)

$$y(0) = c + \frac{1}{1+a_1} \quad \text{--- (2)}$$

equating (1) and (2)

$$1 - a_1 y(-1) = c + \frac{1}{1+a_1}$$

$$c = \frac{1}{1+a_1} - 1 - a_1 y(-1)$$

$$\therefore c = \frac{a_1}{1+a_1} - a_1 y(-1)$$

$\therefore \; y_{zi} = -a_1 y(-1) + \frac{a_1}{1+a_1}$

Total solⁿ

$$y(n) = -a_1 y(-1) + \frac{a_1}{1+a_1} + \frac{1-(-a_1)^{n+1}}{1+a_1}$$

DFT
DISCRETE FOURIER X'FORM

It is a finite duration discrete frequency sequence which is obtained by sampling one period of fourier x'form.

Sampling is done at N equally spaced points over the period points over the period extending from $\omega = 0$ to $\omega = 2\pi$.

DFT of $x(n)$ is $X(K)$. given by

$$\boxed{X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi kn/N}}$$

Here $K = 0, 1, 2, 3 \ldots N-1$

since Submission is for N-points thats why called as N-point DFT.

Inverse DFT

$$\boxed{x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j2\pi nK/N}}$$

where $n = 0, 1, 2 \ldots N-1$

This is N-point IDFT

TWIDDLE FACTOR

$$\boxed{W_N = e^{-j2\pi/N}}$$

This is called as Twiddle factor.

→ Twiddle factor makes the computation easy and fast.

Using twiddle factor
DFT is written as

$$\boxed{X(K) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{kn}}$$

and IDFT is written as

$$\boxed{\underset{x(n)}{IDFT} = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, W_N^{-Kn}}$$

Example :- obtain DFT of unit impulse $\delta(n)$.

Soln :- Acc. to definition of DFT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi Kn/N}$$

Here $x(n) = \delta(n)$

But $\delta(n) = 1$ only at $n = 0$

Thus eq-n becomes

$$X(K) = \delta(0)\, e^{0} = 1$$

$$\delta(n) \xleftrightarrow{DFT} 1$$

SUNDAY 2

NOTES

APRIL 2008 / MAY 2008

Example:- Compute N- point DFT of the following exponential sequence.

$$x(n) = a^n u(n) \quad \text{for } 0 \le n \le N-1$$

Sol:- Acc to Def. of DFT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N} \quad \text{—(1)}$$

Here $x(n) = a^n u(n)$.

$u(n) = 1 \quad$ for $n \ge 0$

$\therefore \quad x(n) = a^n$.

Put value of $x(n)$ in eq=n (1)

$$X(K) = \sum_{n=0}^{N-1} a^n e^{-j2\pi kn/N}$$

$$\text{or } X(K) = \sum_{n=0}^{N-1} (a e^{-j2\pi k/N})^n \quad \text{—(2)}$$

standard Submission formulae

$$\sum_{K=N_1}^{N_2} A^K = \frac{A^{N_1} - A^{N_2+1}}{1-A}$$

Here $N_1 = 0$

$N_2 = N-1$

NOTES

and $A = a e^{-j2\pi K/N}$

$$\therefore \quad X(K) = \frac{\left(ae^{-j2\pi k/N}\right)^0 - \left(ae^{-j2\pi k/N}\right)^{N-1+1}}{1 - ae^{-j2\pi k/N}}$$

$$X(K) = \frac{1 - a^N e^{-j2\pi k}}{1 - ae^{-j2\pi k/N}} \quad \text{--- } (3)$$

Use Euler's Identity to the Numerical term

$$e^{-j2\pi k} = \cos 2\pi k - j \sin 2\pi k$$

$$\boxed{K = \text{Integer}}$$

$$\cos 2\pi k = 1$$

and $\sin 2\pi k = 0$

$$\therefore \quad X(K) = \frac{1 - a^N}{1 - ae^{-j2\pi k/N}}$$

$$a^n u(n) \xleftrightarrow{\text{DFT}} \frac{1 - a^N}{1 - ae^{-j2\pi k/N}}$$

CYCLIC PROPERTY OF TWIDDLE FACTOR.

Twiddle factor

$$W_N = e^{-j2\pi/N}$$

Now the discrete time sequence $x(n)$ can be denoted by x_N

where N = N-point DFT
Range of n = 0 to N-1

x_N can be represented in matrix form as under

$$x_N = \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix}_{N\times 1}$$

DFT of $x(n)$ is denoted by $X(K)$
$X(K)$ can be written as X_K in matrix form

$$X_K = \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{bmatrix}_{N\times 1}$$

this is N×1 matrix
K varies from 0 to N-1

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Acc to Def of DFT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

We can also represent W_N^{kn} in matrix form.
K varies form 0 to N-1 and
n varies from 0 to N-1

$\therefore$

$$W_N^{kn} = \begin{array}{c|ccccc} & n=0 & n=1 & n=2 & \cdots & n=N-1 \\ \hline K=0 & W_N^0 & W_N^0 & W_N^0 & \cdots & W_N^0 \\ K=1 & W_N^0 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ K=2 & W_N^0 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & & & \\ K=N-1 & W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)^2} \end{array}_{N\times N}$$

→ each value is obtained by taking multiplication of k and n.

e.g - if K=2
n=2

Then $W_N^{kn} = W_N^4$.

Thus DFT can be represented in matrix form

$$\boxed{X_N = [W_N]\, x_N}$$

IDFT

$$\boxed{x_N = \frac{1}{N}[W_N^*]\, X_N}$$

NOTES

$W_N^* =$ conjugate of W_N.

*** W_N posseses the periodicity property.

$\Downarrow$

means after ~~every~~ some period the value of W_N repeats.

08.00

09.00

10.00

Let us consider 8-pt DFT

means $N=8$.

11.00

We have $W_N = e^{-j2\pi/N}$

12.00

$\therefore \quad W_N^{kn} = e^{-\frac{j2\pi}{N} \times kn}$

01.00

But $N=8$

02.00

$W_8^{kn} = e^{-j2\pi kn/8_4}$

03.00

$W_8^{kn} = e^{-j\pi kn/4}$

04.00

Now let us obtain W_8^{kn} by putting different values of kn.

05.00

Some calculation can be done by using Euler's Identity.

06.00

value of kn	W_8^{kn}	value of the phasor
0	$W_8^0 = e^0$	1
1	$W_8^1 = e^{-\frac{j\pi}{4} \times 1}$	$0.707 - j\,0.707$

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NOTES

2 $\quad W_8^{2} = e^{-j\frac{\pi}{4}\times 2}$ $\quad 0 - j1$

$= e^{-j\frac{\pi}{2}}$

08.00

09.00

10.00

11.00